Stephen Sexauer
Chief Investment Officer
Allianz Global Investors Solutions

Stephen Sexauer has been the Chief Investment Officer of AGI Solutions since inception in June, 2008, and has been a Managing Director of Allianz Global Investors of America LLC or one of its subsidiaries since May, 2003. Mr. Sexauer is also a member of the AGI Fund Management Board. Mr. Sexauer holds an MBA from the University of Chicago with concentrations in economics and statistics and a BS from the University of Illinois in economics.

Paul Pietranico, CFA
Senior Vice President
Portfolio Manager and Director of Research

Paul Pietranico, CFA has been a senior portfolio manager with AGI Solutions since inception in June, 2008. Mr. Pietranico, with Stephen Sexauer, is directly responsible for portfolio risk profiles, asset allocation and fund selection. He joined Allianz Global Investors of America L.P. in June 2005 as director of the due diligence, investment risk analysis and performance reporting teams. Mr. Pietranico holds a BS in physics, an MA in philosophy of science and an MS in Engineering Economic Systems and Operations Research from Stanford University.

Laurence B. Siegel
Research Director, Research Foundation of CFA Institute

Laurence B. Siegel is research director of the Research Foundation of CFA Institute and a senior advisor to Ounavarra Capital in New York. Until July 2009 he was director of research in the Investment Division of the Ford Foundation, where he worked since 1994. Before that, he was a managing director of Ibbotson Associates, an investment consulting firm that he helped to establish in 1979.

The Illusion of Precision in Target-Date Fund Glidepaths

By Stephen C. Sexauer, Paul Pietranico, CFA, and Laurence B. Siegel

Introduction

Target-date funds automatically vary the asset allocation for the participant. Like any asset allocation problem, determining how to change the asset mix over time in a target-date fund is, at its core, a Markowitz-type optimization problem.

In an optimization problem, for any given set of inputs, the result is a point estimate of the desired amount of assets in each asset class. Thus, the problem appears to be able to be solved with precision. This perceived precision can be seen in classic Markowitz efficient frontier diagrams and in the asset allocation lines that show the glidepath in every target-date fund prospectus.

The optimization inputs, however, are themselves statistical estimates rather than precise measures. We thus argue that an optimization solution is a "cloud," so that precision in the desired asset mix and in its variation over time is only an illusion. Markowitz himself, in his seminal 1952 paper, indirectly acknowledges this issue, by stating he had not addressed "the formation of the relevant beliefs on the basis of observation".[1] This is critically important for the allocation to equities, especially for funds designed for participants who are near or in retirement.

We show how the "optimal" mix for target-date funds varies widely when we vary critical variables such as the savings rate and the volatility of risky assets. Given the expected wide variation of "optimal" glidepaths, we conclude that the prudential principle requires sponsors to provide target-date funds that allocate toward the bottom of the cloud, in terms of equity exposure and overall risk. Failure to adhere to this principle could expose sponsors to a credible charge that they did not design the plan in the best interest of the participants.

The theory of lifecycle portfolio choice

The current popularity of target-date funds did not emerge in a vacuum. Financial economists have considered the question of asset selection over the life cycle for generations. A precursor of the modern theory of lifecycle portfolio choice is Milton Friedman's (1957) lifetime income hypothesis, which says that people adjust their spending, not to their current income, but to what they expect to earn on average over their lifetime.[2] To accomplish this, they must obviously be able to shift their consumption across time by borrowing and lending. There is no asset allocation problem to be solved if the only investing is literal lending (that is, investing in riskless assets).

But shifting consumption across time is potentially much more rewarding if it includes risk-taking to earn an expected return higher than that of riskless bonds or bills. A number of prominent economists have asked how people either do or should allocate their investments to effect this time-shift in consumption.

Paul Samuelson (1969), drawing on work by Kenneth Arrow and Robert Merton (all three of them Nobel Prize winners), argued that the age of the investor should not affect his or her ideal asset mix. Samuelson showed persuasively that time diversification does not "work"; that is, the risk of an investment in a risky asset does not diminish as the investor's time horizon lengthens. This finding implies that an older investor should take as much risk as a young one.[3]

This answer is deeply counterintuitive, and it turns out that theory changed so as to be more consistent with common sense and longstanding practice. Bodie, Merton, and Samuelson (1992) added in human capital and showed why portfolio risk should decline as one gets older.[4] They found that young people have a lot of human capital – defined as the present value of expected future income – but little financial capital. Older people have the reverse. Thus, merely to maintain the overall risk level of one's total (human plus financial) capital, one must reduce the risk level of one's financial-capital portfolio over time! To the extent that today's target-date funds, with their glidepaths, have a theoretical basis, this is it.

Given this starting point, building a target-date fund then becomes an engineering problem, grounded in multi-period Markowitz optimization as we said at the beginning. When calculating an optimal glidepath, however, there are at least three issues to address.

One important issue is the substitution of capital market returns for savings: the more you save, the less capital market risk you have to take. Specifying this rate of substitution precisely is a challenging task because the participant is exposed to some possibility of an unacceptable post-retirement income level, even setting aside the special, and costly, case of investing 100% in the riskless asset. The original academic work simultaneously solved for the optimal asset mix and savings rate over time; however contemporary analyses tend to assume a savings rate and conditionally solve for the asset mix. Ignoring the interdependence between the optimal asset mix and the optimal savings rate can result in excessively risky asset mixes. This issue of trading off saving today for uncertain capital market returns will be treated in a companion essay.

A second issue is that the determination of a glidepath depends on which asset-liability model you select. Among the competing models are:

- Liability-driven investing (LDI), which is usually based on the principle of duration matching;
- An asset-only approach, where the objective is to maximize terminal wealth, subject to a concern about risk;
- The Sharpe model, which minimizes the likelihood of not achieving a minimum acceptable wealth or income level in retirement;[5]
- An explicit consumption or modified LDI model, as proposed in, among other works, a book by Ibbotson, Milevsky, Cheng, and Zhu.[6]

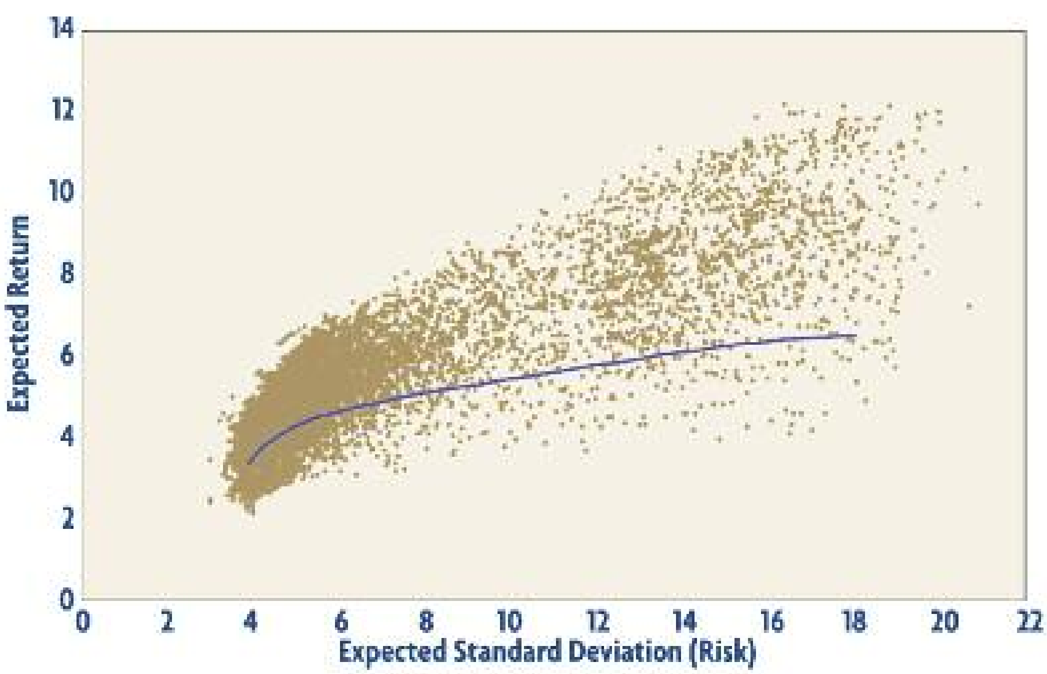
The third major issue is the illusion of precision in the target-date asset mixes. The point estimates look like exact measures of the appropriate percentage in each asset, but they're not. It is this issue we are pursuing in this essay.

The illusion of precision

The idea that Markowitz optimization produces a "cloud" or "fuzzy frontier" is at least a quarter-century old. As the financial economist Philippe Jorion argued in 1985, optimization inputs are estimates or approximations, not exact measurements.[7] Therefore, the outputs are also approximations.

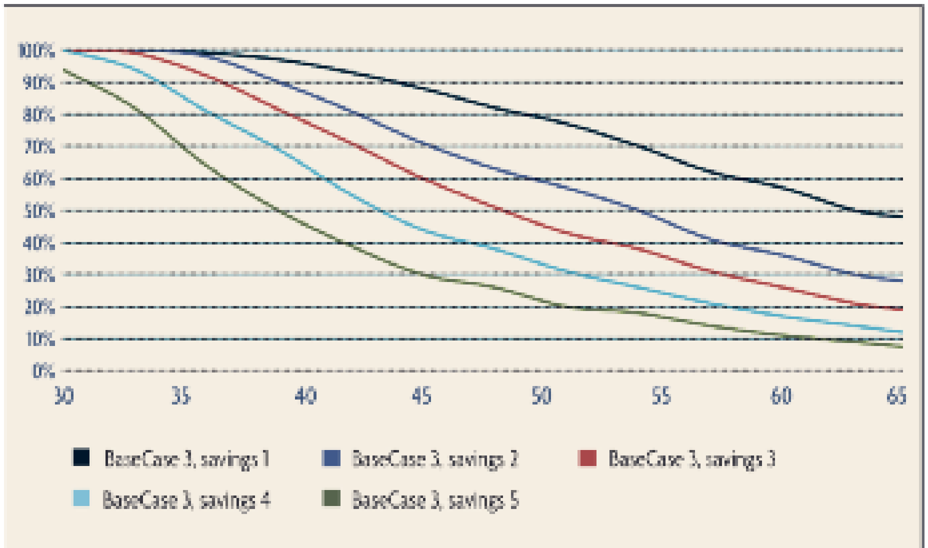
Take the equity risk premium, for example. Current estimates of the premium vary from 1% to 7% with most practitioners using an estimate between 3% and 4%. So if you are using an equity risk premium estimate of 3.5%, it would be more honest to call the estimate 3.5% ± 0.5% (or some larger plus-or-minus number). The resulting allocation to equities, all other things held equal,

Figure 1: Efficient Frontier of Major Asset Classes Portrayed as a Cloud



Source: Calculated by the authors using data in Appendix A.

FIGURE 2: OPTIMAL ASSET MIXES BY PARTICIPANT AGE, WITH VARIATION IN SAVINGS RATE



	Case 1	Case 2	Case 3	Case 4	Case 5
Multiple of Base Case Savings Rate	0.50	0.80	1.00	1.30	1.50

Source: Allianz Global Investors Solutions, risklab GmbH. Appendix B provides technical information on the analysis.
Note: Optimal glidepaths for different investor savings rates during accumulation. The five cases respectively correspond to 50%, 80%, 100%, 130% and 150% of the base case savings rate. The base case savings rate is as specified in Hewitt [2006].[8] The highest savings rate results in the most conservative glidepath with less than 10% in equities at retirement.

will naturally also vary within some range. Apply the same logic to the other inputs, including the other expected or mean returns as well as expected standard deviations and expected correlations, and you get a cloud of plausibly optimal asset mixes, not a single and clearly specified efficient frontier. Although there are techniques to address this specific type of uncertainty in Markowitz-type optimizations, for example, resampling or robust optimization, we want to emphasize that uncertainty in inputs–and there will always be some parameters for which the uncertainty cannot be incorporated into the optimization–results in a cloud of possible solutions rather than single precise points.

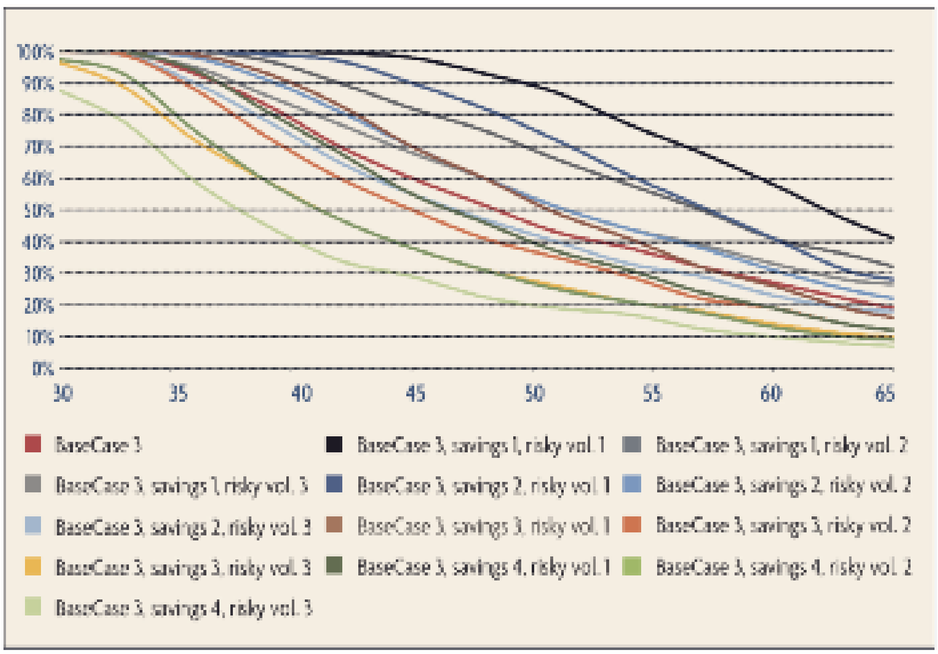
Figure 1 (on previous page) illustrates such a cloud, constructed using current data for five major asset classes: global equities, U.S. REITs, diversified commodity futures, core U.S. bonds, and U.S. TIPS. The cloud has the characteristic that a point within it does not differ in position from another point in the cloud above or below it by a statistically significant amount. An investor, therefore, would not reallocate assets to get from one point to the other with any confidence that the portfolio's expected return per unit of risk had been improved.

How large is the cloud in practice, when allocating assets for target-date funds? Target date glidepaths are often created using techniques that are more sophisticated than classic Markowitz optimization and that adjust for uncertainty in some but not all of the inputs and optimize over multiple time periods. These techniques, nonetheless, possess the same limitations – some inputs are uncertain.

Each input variable that is estimated with uncertainty is a source of "cloudiness." The conventional setup of a target-date asset allocation problem includes inputs for initial financial wealth, human capital, the savings rate, longevity, and bequest motive, as well as the capital market assumptions discussed above. This means there are many moving parts. Figure 2 (at left) shows that by varying just one input, the savings rate, within a three-to-one range, the optimal equity allocation for (say) a 50-year-old man can vary from 20% to 80%, which is consistent with the observed range of allocations at retirement in glidepaths in the industry. And note that we have not even varied the savings rate all that much – individuals' savings rates vary by much more than three to one.

For 140 million American workers, real life is more complicated than just different savings rates. In Figure 3 (below) we vary not only the savings rate

FIGURE 3: OPTIMAL ASSET MIXES BY PARTICIPANT AGE, WITH JOINT VARIATION OF SAVINGS RATE AND RISKY ASSET VOLATILITY



	Case 1	Case 2	Case 3	Case 4
Multiple of Base Case Savings Rate	0.50	0.80	1.00	1.30
Risk Asset Volatility	10%	15%	20%	

Source: Allianz Global Investors Solutions, risklab GmbH. Appendix B provides technical information on the analysis.
Note: Optimal glidepaths for different combinations of the levels of risky asset volatility and investor savings rates during accumulation. The most conservative glidepath, at the bottom of the cloud, is optimal when the investor saves the most and is subject to the highest level of equity volatility.

but also the expected volatility of the risky asset. We vary the volatility within a two-to-one range, which is again quite modest given the actual, historical variation in volatility. With this more realistic but still not complete adjustment, the cloud is now even larger, and makes a mockery of the idea of precision in asset allocation for target-date funds.

Under such circumstances, we must be exceedingly modest about the extent of our knowledge. And when our knowledge is incomplete, the prudent course is to save more and to take less investment risk, especially when participants are near or at retirement – a time when savings balances are high and the time to grow out of losses is limited.

Conclusion

Lifetime savings and investing decisions are complex, and for the average saver, daunting. Lifecycle funds provide a valuable structure to the problem. The most important variable, savings, can be the focus of the worker, while the risk management, asset allocation, and security selection are performed inside the fund by the asset manager. But this solution still leaves open the questions of how much risk should be taken and how risky a given target-date fund is.

We have shown that even those glidepaths that are grounded in theory, having been created with the most powerful optimization techniques available, still produce outcomes with equity allocations for a fifty year old that vary from 20% to 80% – a range so wide as to make it very difficult to sensibly select one glidepath out of the cloud. We conclude by stating three logical consequences of this lack of precision inherent in all target date glidepaths.

The first is that, for the majority of target-date investors who select the default option and rely upon the autopilot nature of target-date program selection, the prudential principle calls for the application of experience and judgment in choosing a glidepath that is biased toward the lower edge of the optimal glidepath cloud in terms of risk level.

The second consequence is that investors would benefit from the fund provider using a public glidepath benchmark that is also near the lower edge of the cloud. A benchmark provides clarity regarding the underlying asset classes, the ability to identify salient risk factors, and a basis for the plan sponsor to make sensible forecasts of fund performance and volatility. Furthermore, a public benchmark makes it possible to apply institutional-quality performance measurement and portfolio attribution techniques.

The third consequence is the ability of the participant to make an informed choice to either default into the prudentially proper lower edge of the cloud, or to make a potentially utility-enhancing decision to add risk by selecting a higher-risk fund or, alternatively, to take away risk by adding cash or selecting a lower-risk fund. This choice is made with the theoretically sound low-risk default choice as a starting point.

Given the vast sums of retirement savings on autopilot in target-date glidepaths, the task of developing the glidepaths should employ capital market theory to the fullest extent possible. But that is a minimum requirement! Making target date glidepaths reliably successful in practice requires full knowledge of their limits, in both theory and practice. Armed with this knowledge, we can make better-informed lifetime savings and investment decisions. ■

Appendix 1

Asset class	Expected return	Expected standard deviation
Stocks	6.0	16.0
REITs	6.5	18.0
Commodity futures	5.0	14.0
Nominal bonds	3.0	4.0
TIPS	4.0	5.0

Correlations (not shown) are calculated using monthly data from March 1997 to July 2011. Asset classes are represented by the following indexes: the Dow Jones Global Total Stock Market Index, the Dow Jones U.S. Select Real Estate Securities Index, the Dow Jones-UBS Commodity Index Total Return, the Barclays Capital U.S. Aggregate Bond Index, and the Barclays Capital U.S. Treasury Inflation Protected Securities Index.

Appendix 2

The optimal glidepaths shown in Figures 2 and 3 were generated from an advanced lifecycle model created by risklab GmbH, an Allianz affiliate based in Munich, Germany. This model uses a stochastic dynamic programming approach over the whole lifecycle (both accumulation and decumulation) and includes both human capital and mortality rates. It aims to minimize the risk of income shortfall in real terms during retirement by finding the optimal allocation between a higher risk asset (4.9% expected return and 18% volatility, i.e., equities) and a lower risk asset (2% expected return and 2% volatility, i.e., short/intermediate-term TIPS). When modeling the accumulation phase, it accounts for stochastic income per Cocco et al. [2005] using data from the Panel Study of Income Dynamics (PSID), human capital as approximated by total discounted future earnings (where the base case assumes a college graduate), and saving rates for different age cohorts based on 401(k) data from Hewitt [2006]. For the decumulation phase, inputs include the income level required during retirement expressed as a percentage of income just prior to retirement (80% of final labor income in the base case), the percentage of required retirement income provided by social programs and pensions (40% of total retirement income in the base case), the National Center for Health Statistics U.S. Life Table from 2004 to directly incorporate mortality probabilities, and a bequest preference factor. Maximum life span is assumed to be 100.

References for Appendix 2:

- Cocco, Joao F., Gomes, Francisco J., & Maenhout, Pascal J. 2005. "Consumption and Portfolio Choice over the Life Cycle." *The Review of Financial Studies*, 18(2), 491–533.
- Panel Study of Income Dynamics, public use dataset. Produced and distributed by the Institute for Social Research, Survey Research Center, University of Michigan, Ann Arbor, MI.
- [U.S.] Centers for Disease Control, National Center for Health Statistics, http://www.cdc.gov/nchs

Footnotes:

1 Markowitz, Harry. 1952, "Portfolio Selection." *The Journal of Finance* Vol. 7, No. 1

2 Friedman, Milton. 1957. *A Theory of the Consumption Function*, Princeton University Press, Princeton, NJ.

3 This argument holds under what we regard as a very peculiar set of assumptions. The investor is presumed to have an embedded risk tolerance, one that does not change as he or she ages. Thus, if the risk of a long-term investment in (say) an equity index is no less than the risk of a short-term one, the investor's allocation to that asset should not change as he or she ages. (We are persuaded that the *investment itself* does not become less risky as one holds it for longer periods, but we are talking about the *investor*. Samuelson does not, to our satisfaction, explain why investors cannot or should not change their minds about risk as they progress through life.) See Samuelson, Paul A. 1969. "Lifetime Portfolio Selection by Dynamic Stochastic Programming." *Review of Economics and Statistics* 51, 239-246; Arrow, Kenneth J. 1965 [1971], "The Theory of Risk Aversion," Reprinted in *Essays in the Theory of Risk Bearing*, Markham Publishing Co., Chicago, 1971, pp. 90-109; Merton, Robert C. 1969. "Lifetime Portfolio Selection Under Uncertainty: The Continuous Time Case," *Review of Economics and Statistics* 51, 247-257; Merton, Robert C. 1971. "Optimum Consumption and Portfolio Rules in a Continuous-Time Model." Journal of Economic Theory 3, 373-413.

4 Bodie, Zvi, Robert C. Merton, and William Samuelson. 1992. "Labor Supply Flexibility and Portfolio Choice in a Life-Cycle Model." *Journal of Economic Dynamics and Control* 16, 3-4 (July-October): 427-449.

5 This model has been modified somewhat. To the best of our knowledge, Sharpe's latest thinking is in Sharpe, William F., Jason S. Scott, and John G. Watson. 2007. "Efficient Retirement Financial Strategies," on William Sharpe's web site at http://www.stanford.edu/~wfsharpe/retecon/ERFS.pdf (July).

6 Ibbotson, Roger G., Moshe A. Milevsky, Peng Chen, CFA, and Kevin X. Zhu. 2007. *Lifetime Financial Advice: Human Capital, Asset Allocation, and Insurance*. Research Foundation of CFA Institute, Charlottesville, VA (April).

7 Jorion, Philippe. 1985. "International Portfolio Diversification with Estimation Risk." *Journal of Business*, vol. 58, no. 3.

8 The base case intertemporal before-tax savings rates were taken from the table entitled, "Contribution Level—By Age" on page 18 of Hewitt. 2006. *Research Report: How Well Are Employees Saving and Investing in 401(k) Plans?* 2006 Hewitt Universe Benchmarks, Hewitt Financial Services.