FAJ
Financial Analysts Journal
Volume 60 • Number 5

TIPS, the Dual Duration, and the Pension Plan

Laurence B. Siegel and M. Barton Waring

By defining "duration" as the sensitivity of an asset's price to changes in some other variable, one may characterize any asset as having an inflation duration, D_i, and a real-interest-rate duration, D_r. Unlike nominal bonds, for which $D_i = D_r$, inflation-linked bonds, such as Treasury Inflation-Indexed Securities (commonly called TIPS), have different values for D_i and D_r. Defined-benefit pension liabilities also have different values for D_i and D_r. Such liabilities can be modeled as bonds (or portfolios of bonds and equities or other assets) held short. Thus, by appropriately combining TIPS and nominal bonds, a manager can build a portfolio that has the same inflation duration and real-interest-rate duration as the liability stream. Equities also have different values for D_i and D_r, so the interaction of equities with TIPS and nominal bonds can be exploited in forming efficient pension portfolios—particularly in defeasing various liability streams.

Not long ago, when the real yield (that is, the yield before adding the inflation adjustment) on Treasury Inflation-Indexed Securities (formerly called Treasury Inflation-Protected Securities, TIPS) exceeded 4 percent, investors could be forgiven for thinking that TIPS were the magical asset. At that time, it was not clear whether equities (much less nominal bonds) offered prospective real returns higher than 4 percent. So, TIPS offered not only inflation protection and the repayment of nominal principal (if one bought TIPS that were selling close to par) but a rich absolute return.

Investors did not figure out the special attractiveness of TIPS for a while after the initial issuance of these securities, but apparently they did so in droves beginning in early 2000: The real yields on these instruments fell from a high of 4.36 percent in January 2000 to a low of 1.03 percent in March 2004.[1] At today's more modest real yields, the case for TIPS is not one of timing or undervaluation but of structural advantage. One should ask: What are the characteristics of this instrument that make it fundamentally different from other securities? Who is the natural clientele for these characteristics; that is, for what kinds of investors are TIPS so attractive that it is rational for them to outbid others to acquire TIPS? Conversely, what kinds of investors should avoid TIPS?

Laurence B. Siegel is director of policy research in the Investment Division of The Ford Foundation, New York. M. Barton Waring is managing director and head of the Client Advisory Group at Barclays Global Investors, San Francisco.

To answer these questions, we focus on the fact that, as Leibowitz, Sorensen, Arnott, and Hanson (1989) pointed out, an asset or stream of cash flows (in their case, equities) can be regarded as having *two* durations—(1) an inflation duration, D_i, or sensitivity of the asset's return to a change in the inflation rate, and (2) a real-interest-rate duration, D_r, or sensitivity of the asset's return to a change in the real interest rate. Although this distinction can, in principle, be drawn with nominal bonds, for such a bond, D_i and D_r are essentially equal to one another and also equal to the regular, or nominal, duration. The distinction between D_i and D_r becomes interesting when applied to TIPS, for which D_i is emphatically not equal to D_r, and to other assets, to liabilities, and to portfolios with the same characteristic.

After exploring this "dual duration" characteristic of TIPS, we draw on Goodman and Marshall's (1988) observation that pension liabilities also have such a dual duration. We develop this parallel to show how TIPS can be used, together with nominal bonds, to hedge the inflation and real-interest-rate risks of pension liabilities. Finally, noting that equities also have an inflation duration that is different from their real-interest-rate duration, we address the question of how to manage the asset class exposures in pension plan, foundation, and endowment fund portfolios and in the investment programs of individuals.[2]

Because TIPS play into investors' natural desire to hedge against inflation and because of economists' special fondness for a security that provides a direct market measure of the real rate of interest, TIPS have received a great deal of attention from researchers in a short time. Yet, there is little evidence that TIPS, with their special qualities (particularly their dual durations), have been incorporated into the standard toolbox of investment strategists. Siegel (2001), drawing on conversations with Waring and others, noted that the dual durations of TIPS are parallel to those of a liability, and Phoa (2001) noted that the duration of TIPS needs to be treated differently from that of nominal bonds.[3] Waring (2000b) showed the connection between the dual duration of certain assets (including TIPS) and liabilities, and he discussed the natural hedging opportunities this characteristic generates. We address this connection and opportunity in this article.[4]

Two Durations

To define the principal terms of our discussion, we must first note the "Fisherian" decomposition of nominal interest rates, n:[5]

$$\left(1+\frac{n}{f}\right) = \left(1+\frac{i}{f}\right)\left(1+\frac{r}{f}\right), \tag{1}$$

where

i = inflation rate expected by investors over the life of the bond

r = real interest rate

f = compounding frequency or number of payments a year ($f = 1$ for bonds that pay an annual coupon; $f = 2$ for bonds that pay semiannually)

The linear approximation of Equation 1, the form we generally use, is

$$n = i + r. \tag{2}$$

The *modified duration* of a bond, D_n, measures the expected or forecast sensitivity of a bond to changes in its nominal yield to maturity.[6] Specifically, modified duration is the percentage change in price for a unit change in nominal yield:

$$D_n = -\left(\frac{1}{P}\right)\left(\frac{\partial P}{\partial n}\right), \tag{3}$$

where P is the price of the bond.

For a zero-coupon bond, the duration is close to being equal to the bond's term to maturity, t:[7]

$$D_n = \frac{t}{1+n}. \tag{4}$$

Durations for a Nominal Bond. For a nominal bond, the inflation duration and real-interest-rate duration can be separately defined, but (as the reader will see) they are equal, or almost equal, to each other and to the nominal duration.

The concept of modified duration, defined in Equation 3, can easily be extended to expected inflation and real interest rates by defining inflation duration D_i as the percentage change in price for a unit change in inflation:[8]

$$\begin{aligned} D_i &= -\left(\frac{1}{P}\right)\left(\frac{\partial P}{\partial i}\right) \\ &= \left(1+\frac{r}{f}\right)D_n, \end{aligned} \tag{5}$$

The real-interest-rate duration is the percentage change in price for a unit change in the real interest rate:

$$\begin{aligned} D_r &= -\left(\frac{1}{P}\right)\left(\frac{\partial P}{\partial r}\right) \\ &= \left(1+\frac{i}{f}\right)D_n. \end{aligned} \tag{6}$$

If all yields n, r, and i are small, then

$$D_n \approx D_i \approx D_r. \tag{7}$$

The reader may be surprised to see that these three durations are not *exactly* equal even though we are discussing a *nominal* bond; that they are not equal is an artifact of the somewhat obscure calculus used to define duration. In practice, the three durations may as well be equal because a change in the interest rate has the same effect on a nominal bond's price no matter whether the interest rate change arises from changes in inflation or changes in the real interest rate. Thus, investors (reasonably) behave as if

$$D_n = D_i = D_r. \tag{8}$$

In other words, using nominal bonds, the investor cannot bet separately on changes in inflation and changes in real interest rates; that is, the investor cannot hedge inflation and real-interest-rate risks independently of one another.

Bond traders and analysts often acknowledge changes in inflation and changes in real interest rates as separate sources of price movements in nominal bonds. Because these two variables are invisibly combined to arrive at the nominal yield, without the component parts being observed, however, the existence of the two *durations* for nominal bonds is rarely recognized. This omission is perfectly acceptable because identifying the two durations adds little analytical value for nominal bonds. The story is different for inflation-linked bonds and for many other assets.

Durations for an Inflation-Linked Bond. Based on Waring (2000b), the price or present value (PV) equation for a stylized, prototypical zero-coupon bond fully indexed to inflation (with no embedded put option) is

$$P = \frac{F(1+i)^t}{(1+i)^t(1+r)^t}. \quad (9)$$

Thus, the face or par value, F, increases at the inflation rate rather than remaining level over time as in a conventional bond.[9] Because inflation rate i is in both the numerator and denominator of Equation 9, the price of such a bond does not change with respect to changes in the rate of inflation—only with respect to changes in real rates. The durations with respect to i and r become

$$D_i = 0 \quad (10)$$

and

$$D_r = \frac{t}{1+r}. \quad (11)$$

Equation 9 can be generalized to describe coupon-paying inflation-linked bonds (we will continue to ignore the put option):[10]

$$P = \sum_{j=1}^{t} \frac{C(1+i)^j}{(1+i)^j(1+r)^j} + \frac{F(1+i)^t}{(1+i)^t(1+r)^t}, \quad (12)$$

where C is the initial coupon, which inflates at inflation rate i.

The duration with respect to i remains zero, but the duration with respect to r is:

$$D_r = -\left(\frac{1}{P}\right)\left(\frac{\partial P}{\partial r}\right), \quad (13)$$

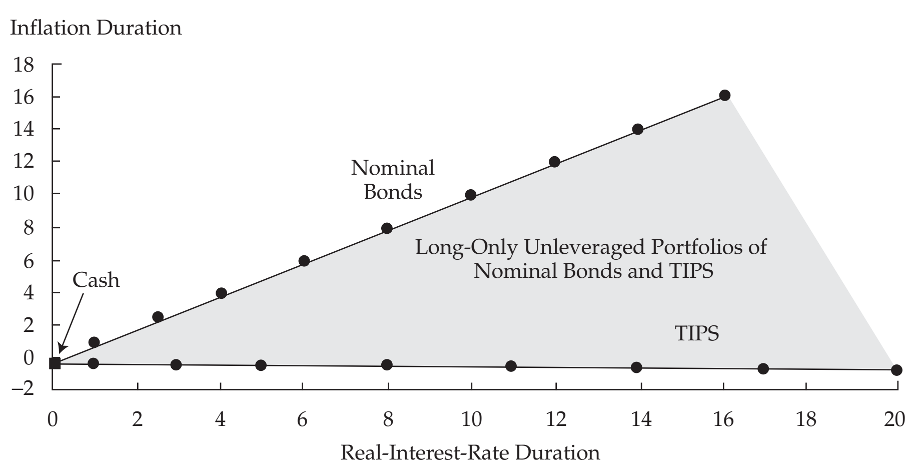
which in typical circumstances is a large negative number, although smaller in absolute value than D_r in Equation 11.[11] (The duration of an inflation-linked bond with respect to r is large because of the low levels of real yields.)

This dual duration will form the basis for an asset/liability planning approach that is completely different from, and much better than, anything that can be achieved with only nominal bonds and nonbonds (say, cash and equities).

Interestingly, most of the prior research on the duration of TIPS ignores their inflation and real-interest-rate durations and focuses on methods of estimating their *nominal* duration.[12] The reason is that the nominal cash flows, and hence the nominal duration, of TIPS are uncertain whereas nominal bonds have a duration known with certainty in advance.

Graphing Dual Duration. Thus, all nominal and inflation-linked bonds (and liabilities modeled as nominal or inflation-linked bonds or portfolios of them) have two durations, which can be visualized as ordered pairs, as shown in **Figure 1**. Nominal bonds are represented by the "family" of points on the line extending from the origin at a

Figure 1. Inflation Duration and Real-Interest-Rate Duration of Nominal Bonds and TIPS



roughly 45° angle; TIPS are represented by a different family of points—those on the line that is almost parallel to the x-axis.

When nominal and inflation-linked bonds are combined in a portfolio of bonds, the real-interest-rate duration of the portfolio will be a weighted average of the constituent real durations:

$$D_r(port) = \left(\frac{1}{P(port)}\right)\left(\frac{\partial P(port)}{\partial r}\right) \tag{14}$$
$$= \frac{P(nom)}{P(port)} D_r(nom) + \frac{P(TIPS)}{P(port)} D_r(TIPS).$$

The inflation duration of the portfolio will be simply the weighted duration of the nominal bond, because the inflation-linked bond has an inflation duration of zero:

$$D_i(port) = \left(\frac{1}{P(port)}\right)\left(\frac{\partial P(port)}{d_i}\right) \tag{15}$$
$$= \frac{P(nom)}{P(port)} D_i(nom).$$

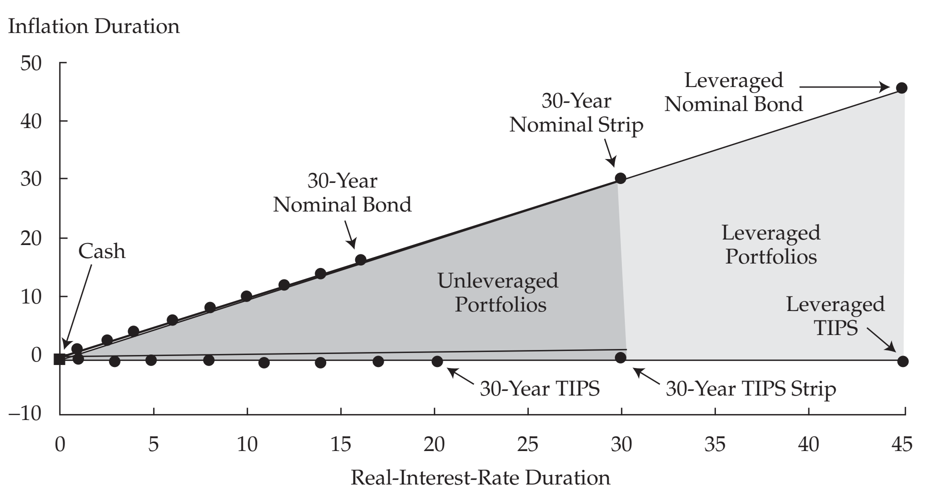
The nominal duration of the portfolio will be between the two, but the notion of nominal duration loses its usefulness when real-rate duration and inflation duration are no longer equal. The reaction of the portfolio to a given change in yield will depend on the source of the change: If a change in nominal yield is entirely a result of changes in the real yield or real interest rate, then $D_n(port) = D_r(port)$; if the change in nominal yield is entirely a result of changes in inflation expectations, then $D_n(port) = D_i(port)$.

As Kothari and Shanken (2000) pointed out, the empirical duration of TIPS has been slightly less than the zero value that is predicted in Equation 12, apparently because investors have reacted to unexpected inflation by increasing their demand for inflation hedges, including TIPS. In other words, an increase in inflation has caused the real interest rate to decrease slightly and caused TIPS to rally. This effect is represented by the gentle downward slope of the TIPS line in Figure 1. Because this second-order historical effect might not be repeated in the future, the TIPS line could have been drawn directly on top of the x-axis.

The shaded area between the two lines in Figure 1 is totally occupied by the duration of unleveraged, long-only portfolios of nominal bonds and TIPS.

Figure 2 echoes the lines for duration of nominal bonds and for TIPS and shows that by using "stripped" (principal-only) nominal bonds and TIPS, one can extend each line upward and to the right, thus expanding the space that can be reached through unleveraged, long-only portfolios of nominal bonds and TIPS, including strips. (We consider that the use of strips does not constitute leverage.) Leveraged portfolios lie in the light grey area, and unleveraged portfolios lie in the darker area. Thus, Figure 2 also shows that if one can leverage at the riskless rate (so that the leveraging asset is at the origin), the space that can be reached through long-only investing increases—going beyond the strips space shown in dark grey and extending infinitely to the right.[13] Portfolios with long durations such as shown in Figure 2 are interesting because of the long durations of many pension liabilities.

Figure 2. Unleveraged and Leveraged Long-Only Portfolios of Nominal Bonds, TIPS, and Strips



Finally, **Figure 3** shows that short selling of nominal bonds or TIPS or both when combined with the use of leverage makes any desired combination of D_i and D_r achievable. (Without TIPS, only the points on the nominal-bond line are achievable.) Of course, nobody really invests this way, but the conclusion is too appealing to pass over without mentioning.

Dual Duration and the Defined-Benefit Plan

Most defined-benefit (DB) plans annually pay the retiree a percentage of "final pay" (the compensation in the last year that the employee worked) or, more typically, a percentage of "final average pay" (an average of the compensation paid in the last few—say, three to five—years the employee worked). The percentage of final average pay that the retiree receives is typically a function of the number of "years of service" the employee provided; thus, if the payout formula is 2 percent per year of service and the employee has worked at the company for 30 years, he or she retires with an income equal to 60 percent of final average pay.

This amount may be increased after retirement by a cost of living adjustment (COLA) to give retirees a fixed *real* income. Almost all corporate plans are structured without COLAs because contractual COLAs increase the cost of the plan. Most public plans and many plans sponsored by nonprofit organizations, such as universities, foundations, and churches, have contractual COLAs.

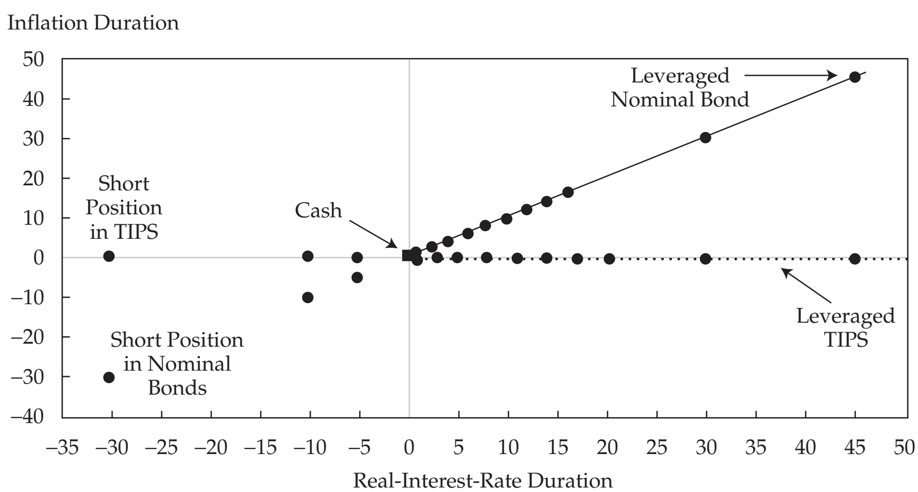
The growth path between the current level of compensation and final average pay is, of course, at least partially determined by generalized inflation, which is relatively highly correlated with wage inflation.[14] (Other factors include merit, seniority, and promotional increases in pay, which are proxies for increases in productivity.) Thus, even in plans that do not have COLAs, the liabilities have significant exposure to inflation risk and need to be defeased by assets that do well when unexpected inflation occurs. (Unexpected inflation is inflation that is not embedded in nominal-bond yields and other asset prices.) Suitable assets include TIPS, cash, and potentially, equities and real estate.

Model of Pension Liability. To construct a simple model of the pension plan's liability, we begin with a plan that lacks a COLA and then discuss a model that takes a COLA into account.

Pension liabilities with no COLA. To analyze the inflation and real-interest-rate durations of DB pension plans, we use the framework established by Goodman and Marshall. They used stylized one-participant examples to illustrate the impact of inflation on pensions. The participant is assumed to have worked for the sponsor for 20 years and to have 10 years to retirement. Goodman and Marshall described other details of the example as follows:

> While the worker is older than the average member of the plan, he is representative for purposes of calculating the typical pension plan liability, because benefit obligations are weighted by length of service. The plan expects to pay the representative individual for 17 years after retirement. [Payout] is based on a formula of 2 percent of his final salary per year of service. There is no [COLA]. . . . Our analysis uses the projected benefit obligation (PBO) as a proxy for the economic liability.

Figure 3. Leveraged Long–Short Portfolios of Nominal Bonds and TIPS



The PBO, an actuarially derived measure of pension liability, measures the liability of the pension plan conditional on the sponsor being a going concern but without accounting for pension liabilities associated with future hiring.[15] Unlike the accumulated benefit obligation, which is closely related to the liability that would be incurred if the company were to close its doors today and pay pensions based only on employees' past service, the PBO includes service accruals and salary inflation from the present through the time when benefits are to be calculated on the basis of final pay or final average pay.[16]

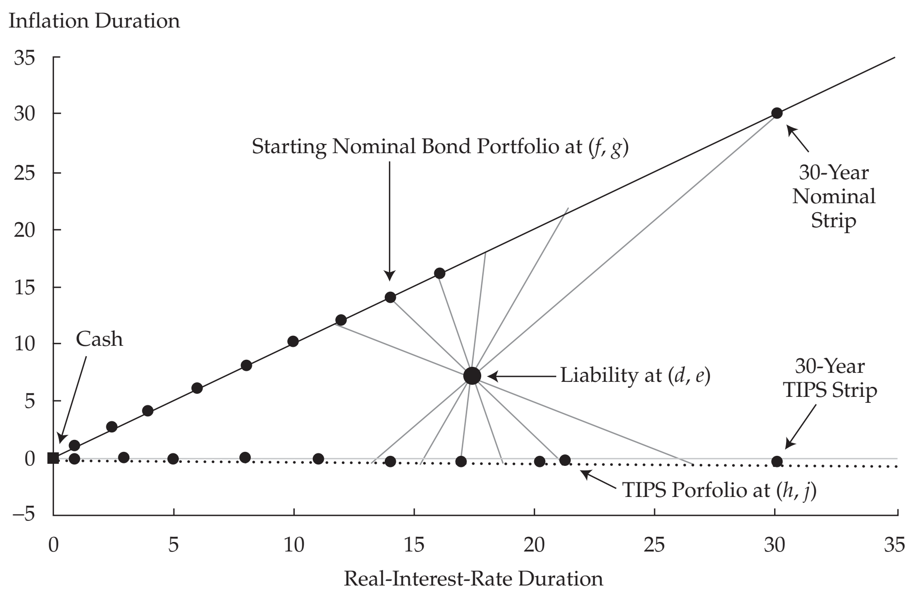
Goodman and Marshall, using the high interest rates of their time as a starting point, constructed a complex inflation model that differentiates between "anticipated" and "unanticipated" inflation. They found that the liability for this stylized single participant had an inflation duration of 5.5 and a real-interest-rate duration of 15.5.[17] We recalculated these durations with more up-to-date inflation and real interest rates (3 percent and 2 percent, respectively) as the starting point and with no distinction between expected and unexpected inflation. We found an inflation duration, D_i, of 7.6 and a real-interest-rate duration, D_r, of 17.5 for the same hypothetical participant. In other words, the present value of the liability *decreases* by 7.6 percent in response to a 1 percent increase in the inflation rate.[18] The reason for the decrease is that the unexpected inflation causes a 1 percent rise in the nominal discount rate but less than a 1 percent rise in the nominal payments to the retiree (because inflation shocks are transmitted to the liability only through the point of retirement).

Figure 4 maps this result in dual-duration space. No single asset, or even a portfolio of nominal bonds, has an inflation duration of 7.6 and a real-interest-rate duration of 17.5. But Figure 4 shows a whole family of portfolios of TIPS and nominal bonds that do—that is, the family of portfolios shown as a "star" of lines passing through the point representing the liability, point (d,e). Consider, for example, a nominal-bond portfolio having real-interest-rate duration f and inflation duration g—that is, the portfolio at coordinates (f,g) in Figure 4 (note that f is approximately equal to g). The line must also pass through the liability, which in the present example is at coordinates (17.5, 7.6) but which we have generalized as (d,e). This line is then described by

$$D_i = \left(\frac{g-e}{f-d}\right) D_r + \left(\frac{ef-dg}{f-d}\right). \quad (16)$$

The portfolio of TIPS that one must combine with the starting nominal-bond portfolio to arrive at the liability-defeasing portfolio is the point at

Figure 4. Portfolios of Coupon-Paying and Stripped Nominal Bonds and TIPS that Defease Pension (without a COLA) Liability



which the line in Equation 16 intersects the TIPS line, point (h,j). This point is given by the exuberant expression:

$$(h,j) = \left[\frac{c-(ef-dg)/(f-d)}{(g-e)/(f-d)-z}\right], \left(\frac{c(g-e)/(f-d)-z(ef-dg)/(f-d)}{(g-e)/(f-d)-z}\right), \quad (17)$$

where z is the slope of the TIPS line (a small negative number in the present example) and c is the intercept of the TIPS line (zero in the present example).

The weight of TIPS in the liability-defeasing portfolio is thus given by the length of the line segment from (f,g) to (d,e) divided by the length of the line segment from (f,g) to (h,j). The expression for this weight is even more exuberant, so we leave it out, but it is easy to calculate numerically because these line lengths are simply Euclidean distances, the general form of which is the familiar $\sqrt{x^2+y^2}$.

Given a starting portfolio of nominal bonds with a real-interest-rate duration of 15 years and the liability that appears in Figure 4, the liability-defeasing portfolio that is found by our analysis is 48.4 percent TIPS, where the TIPS have a real-interest-rate duration of 20.17 years. This weight in TIPS is quite large, at least when compared with the weights in most current pension plans. Thus, the seemingly paradoxical result is that even a pension plan whose liability *declines* with an increase in inflation obtains a benefit from adding TIPS.

Goodman and Marshall went to great lengths to design portfolios of nominal bonds that best hedged the interest rate risk of the liability. Although the hedge they designed is optimal, in the sense of there being no better hedge available with only nominal bonds (or nominal bonds and cash), it is nevertheless imperfect. By mixing in TIPS and TIPS strips, which had not yet been issued when Goodman and Marshall did their work, we fully hedge *both* the inflation risk and the real-interest-rate risk of the liability.[19]

■ *Pension plans with a COLA*. When the assumptions in Goodman and Marshall's stylized liability model are kept but a complete COLA equal to the inflation rate is added, the inflation duration of that liability falls to 0 and the real-interest-rate duration rises to 18.1 years.[20] One does not need to conduct the full analysis to see that with a full COLA, the risk-minimizing bond portfolio will consist entirely of TIPS.[21] In practice, COLAs are rarely set equal to U.S. Consumer Price Index inflation. If the COLA is lower than CPI inflation, the inflation duration will approach zero but not reach it.

Building the Total Plan Liability. Starting with the one-participant model, one can construct the liability of the whole pension plan by summing the characteristics of the participants. This method is what actuaries follow. They conduct a census of participants and apply probabilistic forecasting methods to determine the likely number of participants who will reach retirement age with vested benefits, live to specific ages, and so forth. The result is a stream of cash flows that looks like a peculiar bond (one with some uncertain cash flows and an extremely long life) held short by the plan sponsor. The inflation and real-interest-rate durations of this liability can easily be calculated.

Implications

The dual-duration nature of TIPS and the role of these securities in optimal portfolios have important implications for asset/liability management in an overall DB pension plan, for foundation and endowment management, and for the portfolios of individual investors.

Asset/Liability Management of the DB Plan. So far, we have presented a framework for analyzing and hedging the risks of a given pension plan along the dimensions of inflation duration and real-interest-rate duration. If one believes that these two durations are the only risks to which pension plans are exposed and if the plan sponsor is satisfied with the yields offered by nominal bonds and TIPS, we would at this point be done with pension investing: The risks represented by the two durations could be completely defeased (eliminated in an asset/liability matching context) by portfolios of nominal bonds and TIPS in which the proportions of each and the durations of each were determined by the foregoing analysis. Most plan sponsors would consider these yields much too low, however, so they hold equities and other risky assets in an attempt to earn higher overall rates of return on the pension plan. In addition, most sponsors believe that pension plans are exposed to additional risks that are not captured by the dual-duration analysis and are best dealt with by holding equities and other non-fixed-income assets.

Numerous authors have described pension asset/liability management as an optimization problem, with the liability modeled as an asset held short.[22] But optimization is a single-period approach, whereas the dual-duration-matching approach that we have been discussing is inherently multiperiod and is not amenable to the customary risk–return-balancing solutions in mean–variance optimization.[23] In other words, there is no theory—no equivalent to single-period mean–variance optimization—that prescribes how to optimally balance risk against return in duration space (that is, to determine how much additional duration risk to take to obtain the higher expected return on longer bonds). We can try, informally, however, to bring the quite different threads together.

■ *Surplus optimization*. Mean–variance optimization is a strong general approach to deciding what mix of assets to hold. If one models the liability as an asset held short and puts it into the optimizer, one has set up an asset/liability or "surplus"optimization problem.[24] Surplus optimization asks: What is the expected return of the surplus (assets minus liabilities)? What is the expected risk, or volatility, of the surplus? And at each level of surplus risk, what asset mix delivers the highest surplus return?[25]

Like any stream of cash flows, a pension liability may be modeled as a "bundle of betas" (or exposures to various market risk factors) plus an idiosyncratic risk term that is not related to any market factor. Viewed from this perspective, the asset portfolio that produces the least surplus risk is the portfolio having the same bundle of betas as the "liability asset." This portfolio is analogous to the minimum-variance portfolio in asset-only optimization and is not necessarily the portfolio one should hold. If the investor wants to take more risk to try to earn a higher surplus return, then he or she should move upward and to the right on the surplus efficient frontier—in practical terms, hold more equities than are in the minimum-surplus-variance portfolio.

■ *Dual-duration matching in surplus optimization*. The dual-duration-matching approach can be reconciled in two ways with surplus optimization; neither is perfect, but either is likely to be good enough:

1. Treat TIPS and nominal bonds as separate asset classes in the surplus-optimization problem. This setup will cause TIPS and equities to be treated as partial substitutes; as Leibowitz et al. demonstrated, equities, like TIPS, have a relatively short inflation duration but a long real-interest-rate duration. As one moves up and to the right on the surplus efficient frontier—that is, as one increases equity holdings—the weight of TIPS decreases more rapidly than the weight of nominal bonds, which reflects the inflation-hedging property of equities.[26]
2. Group TIPS and nominal bonds into a single asset class ("bonds"), then conduct the surplus optimization. The resulting weight in bonds will be held fixed. The TIPS and nominal bonds within the bond asset class in the resulting bond allocation then need to be reweighted—by bringing in stripped, leveraged, and/or shorted TIPS and nominal bonds if necessary—so that the inflation duration of the total portfolio on the asset side (including equities) is equal to the inflation duration of the liabilities and so that the real-interest-rate duration of the total portfolio on the asset side (including equities) is equal to the real-interest-rate duration of the liabilities. This method sounds like a tall order, but Figure 3 shows that it can be achieved; the strips, leverage, and/or short positions are used to "goose" each duration to the desired level. To achieve this result, one also needs a workable estimate of the duration of an equity portfolio, information that can be obtained only with less-than-perfect accuracy.[27]

Note that the first approach produces a solution that is mean–variance efficient but not perfectly duration matched (in either inflation or real-interest-rate duration). The second solution is completely dual-duration matched but is not mean–variance efficient because the equity and bond weights are held fixed when the optimizer would cause them to vary.

Foundations and Endowments. A number of authors have tried to extend the asset/liability framework from pension fund analysis to endowed institutions, such as foundations and university endowment funds (see, for example, Dybvig 1999). The attempt tends not to work well because the liabilities of most endowed institutions are not fixed like those of pension funds. A private foundation, for example, is required to pay out annually in grants an amount equal to or greater than 5 percent of its then-current assets.[28] Thus, one cannot model this liability independently of the assets, which means that one cannot construct a portfolio of assets that defeases the liability or that minimizes shortfall risk. The strategy adopted by most foundations is to manage the fund as an asset-only portfolio, thus trying to make as much money as possible within the risk tolerance of the institution.

University and other endowment funds, similarly, have no fixed liabilities. They serve the institution by contributing as much as reasonably possible to meeting operating expenses while attempting to maintain "intergenerational equity" or "intergenerational neutrality" by conserving principal and not spending too much in the current time frame.

Endowments are generally *not* subject to a minimum-payout requirement in the way private foundations are, so they have some flexibility to cut spending if, say, asset values are down. The needs of the institutions they serve, however, tend to be negatively correlated with the economy and stock market: When times are tough, donations and tuition receipts are down and the need to spend out of the endowment is greater than ever, so the endowment's ability to lawfully cut payout is not very helpful in practice.

Note that both foundations and endowments are paying out *real* dollars. Their payouts go to purchase goods and services, the cost of which tends to rise with general inflation. Although some of these expenses—in particular, those associated with higher education—have been rising faster than the general inflation rate, the costs can be thought of for investment planning purposes as inflation-indexed bonds held short—similar to a DB pension plan with a full COLA.[29] And as demonstrated earlier, the risk-minimizing portfolio for such a "liability" is 100 percent in TIPS.

By no means do we want to suggest that foundations and endowments hold 100 percent in TIPS. The yields are far too low. A foundation with a 5 percent payout requirement needs to earn a return equal to 5 percent plus the inflation rate to stay whole in real terms—that is, keep the purchasing power of its grants (and its asset value) stable over time. Even the longest-duration TIPS yield only about 2 percent plus the inflation rate, so a TIPS-only portfolio guarantees that the foundation's assets will decline in real value over time. Endowments also need returns that are higher than those offered by TIPS. As with pension plans, therefore, risk minimization is not a fully acceptable strategy for foundations and endowments; they need to take risks (equity risk and, potentially, other types of risk) to meet the basic objectives they were created to achieve. Again as with pension plans, however, the proportion of foundations' and endowments' assets that is ideally invested in TIPS is substantially larger than the proportion now invested in that asset class.

Individual Investors. The typical individual investor's most compelling problem is saving for a potentially long retirement during which the investor's ability to earn additional labor income steadily declines. This proposition is very expensive in comparison with most other personal financial goals, so one can think of the individual investor's challenge simply as being that of funding and administering a one-participant DB pension plan with a full COLA. The "liability" is a stream of real income sufficient to maintain the investor's standard of living after retirement.

In a multiparticipant DB plan, mortality risk (the risk of outliving one's money) is shared and thus effectively eliminated; the plan pays the beneficiary as long as he or she remains alive.[30] In the one-participant plan called "individual investing," however, the investor must either save enough to live to about age 105—or buy an annuity. Thus, the chief difference between individual investing and a DB plan is that the individual must pay the additional cost (either through oversaving or buying annuities) of hedging mortality risk; otherwise, they are much the same, and the same analysis that we applied to DB plans can be applied to individuals' portfolios.[31]

Because the individual's liability stream resembles that of a DB plan with a COLA, the inflation duration is zero. One might, therefore, conclude (in isolation) that TIPS will form a large proportion of the optimal portfolio. But as with DB plans, such is not always the case; TIPS yields are too low. Almost all individual investors, like almost all DB plans, will want to hold an optimized portfolio of low-risk assets (TIPS and other fixed-income securities) and riskier assets (including but not limited to equities). Determination of the "right" portfolio for a given individual ideally involves a full asset/liability study, as it does for a DB plan.[32] Surplus optimization and dual-duration matching can then be applied as we suggested for DB plans; because of the zero inflation duration of the individual's liability, the bulk of the individual's fixed-income assets is likely to be in TIPS, even if equities are also held.[33]

Conclusion

Using the dual-duration analysis, we demonstrated what many observers already know—that TIPS are almost ideally designed for pension, foundation, endowment, and individual tax-deferred use. Different clienteles will wish to hold different weights in TIPS, but almost everyone in these broad categories will want some of them.

Who should avoid TIPS? Anyone with purely nominal liabilities. Insurance companies that have issued only nominal annuities and life insurance policies are an example. Only a few investors are in this category.

If investors behave as we are recommending, TIPS yields (which have fallen) could drop even lower. So, on the one hand, the appeal of TIPS fades at some level of yield; on the other hand, all expected returns are probably lower in our current low-interest-rate environment.

Of course, low yields will also motivate further issuance of TIPS by the U.S. Treasury or of reasonably close substitutes for TIPS by other issuers. Such alternative issuers might be corporate, municipal, and foreign entities. In fact, non-U.S. governments have issued inflation-indexed bonds for much longer than the U.S. Treasury has. Although one cannot directly hedge U.S. inflation by buying bonds that have their cash flows indexed to the inflation rate in another country, one can obtain protection from major global inflationary events, such as oil price shocks, by doing so and simultaneously hedging the issuer's currency back to the U.S. dollar.

Finally, total-return swaps and other derivatives may increase the supply of inflation-hedging instruments somewhat. These securities are useful, however, only to the extent that their issuers can defease or hedge them. The limiting factor for these instruments, then, is the supply of primary inflation-indexed cash flows that are available for repackaging or leveraging.

In conclusion, many categories of investors have a natural appetite for inflation-indexed bonds. This appetite can be expected to grow as the fit between the instruments and investors' needs is widely communicated. The world's issuers of fixed-income securities ought to take advantage of the opportunity provided by low yields and expanding demand in this intriguing asset class.

This article draws on a series of presentations to Barclays Capital Global Inflation-Linked Bond Conferences and the authors' writings (published or in process). We are grateful to John Pirone, who was instrumental in developing the dual-duration-matching solution, and to Thomas S. Coleman and Thomas K. Philips, who did the heavy lifting in making sure that the math was correct and clearly communicated. (Mr. Philips also provided Appendix A.) We thank all who helped us in preparing this article, including Linda B. Strumpf, Susan A. Ollila, Clinton L. Stevenson, Donald J. Galligan, P. Brett Hammond, and Paul D. Kaplan.

Appendix A. Inflation Duration and Real-Interest-Rate Duration Mathematics

Because Equations 5 and 6 are not obvious, we offer the following details.

The price or present value of a nominal bond is

$$P = \sum \frac{C_j}{(1+n)^j}, \tag{A1}$$

where C_j is the bond's cash flow at time j and n is the nominal interest rate or yield.

Duration is defined as the *percentage* change (thus the division by P) in the price of a bond per unit change in some other variable (here, the nominal interest rate, n); multiplication by –1 reflects the fact that bond prices move opposite to their yields:

$$\begin{aligned} D_n &= -\left(\frac{1}{P}\right)\left(\frac{\partial P}{\partial n}\right) \\ &= -\frac{1}{P}\sum \frac{jC_j}{(1+n)^{j-1}}. \end{aligned} \tag{A2}$$

Applying Fisher's decomposition in Equation 1 and setting $f = 1$ produces the inflation duration,

$$\begin{aligned} D_i &= -\left(\frac{1}{P}\right)\left(\frac{\partial P}{\partial i}\right) \\ &= -\frac{1}{P}\sum \frac{jC_j}{(1+i)^{j-1}(1+r)^j} \\ &= \frac{1+r}{P}\sum \frac{jC_j}{(1+i)^{j-1}(1+r)^{j-1}} \\ &= (1+r)\,D_n, \end{aligned} \tag{A3}$$

and the real-interest-rate duration,

$$\begin{aligned} D_r &= -\left(\frac{1}{P}\right)\left(\frac{\partial P}{\partial r}\right) \\ &= -\frac{1}{P}\sum \frac{jC_j}{(1+i)^j(1+r)^{j-1}} \\ &= \frac{1+i}{P}\sum \frac{jC_j}{(1+i)^{j-1}(1+r)^{j-1}} \\ &= (1+i)\,D_n. \end{aligned} \tag{A4}$$

Notes

1. These figures are the real yields on the Salomon Smith Barney Inflation-Linked Securities Index, a market-capitalization-weighted benchmark consisting of all the U.S. Treasury inflation-indexed issues outstanding as of a given date. As of March 2004, the index contained 1 bond (out of 12 in the index) that was priced to have a negative real yield; 3 more were priced to have a real yield of less than 1 percent. The real yield at the long end

of the TIPS yield curve was approximately 1.9 percent as of March 2004.

2. In the general case, D_i and D_r are separate properties of an asset and, typically, have different values, as we find for equities, TIPS, and pension liabilities. A nominal bond, for which D_i equals D_r, is an unusual special case.
3. In Siegel, see pp. 69–70 and Note 7 on p. 70.
4. The dual-duration hedging solution we describe here was developed in connection with client work by Waring and his colleague John Pirone during 1998 and 1999. Waring (2004a) provides more detail on asset/liability hedging through duration matching by using the dual-duration framework set forth here; Waring (2004b) covers surplus optimization and the economic character of the liability. A summary of all these topics is in Waring (2004c).
5. Fisher (1965). The real interest rate, r, may be viewed as composed of the *pure* real interest rate (or time value of money) and various risk premiums (such as the premium investors require for being exposed to uncertain future inflation rates). Further decomposition of r is discussed in D'Vari and Chugh (2002). By "interest rate" in Equation 1, we mean the yield on the bond itself, not interest rates in general. Yield is synonymous with yield to maturity for noncallable bonds, and all of the TIPS covered in this study are noncallable.
6. Macaulay (1938) set forth the original "duration" measure, which is the present-value-weighted average time to receipt of the cash flows from the bond, and which is now called "Macaulay duration." We label this measure D_m, and it is calculated as $D_m = \frac{1}{P}\sum_{j=1}^{t} PV(CF_j) \times j$, where P is the price of the bond, t is the time to maturity of the bond, PV is present value, and CF_j is the cash flow of the bond in period j. Modified duration, D_n, is related to Macaulay duration as follows: $D_m = D_n\,(1 + n)$ for bonds with an annual coupon, and $D_m = D_n\,(1 + n/2)$ for bonds with semiannual coupons.
7. Equation 4 is for a zero-coupon bond with yield compounded annually.
8. Appendix A provides the derivation of Equations 5 and 6.
9. Actual TIPS are somewhat more complex, although conceptually similar; one of the complexities of TIPS is the embedded put option that arises from the fact that TIPS flip into being a nominal bond, and hence a *deflation* hedge, when their price, including accrued inflation adjustments, falls below par.
10. The put option is far from worthless for TIPS with a price close to par in an economic environment in which some observers perceive a real threat of deflation. By ignoring the put option (to incorporate it would make the math very complex), we *understate* the attractiveness of TIPS. Thus, if one is concerned about deflation as well as inflation, TIPS are even more desirable than this analysis indicates.
11. Empirically, $D_i = 0$ will not turn out to be a precise estimate, because a change in inflation affects the real interest rate (and, potentially, vice versa) and because other factors may be at work (Kothari and Shanken 2000).
12. When researching TIPS durations in 1998 and 1999 to determine whether others had observed the dual duration of these instruments, Waring found only one research report out of many dozens reviewed that correctly identified the dual-duration characteristic of TIPS. Most research reports dealt only with the nominal duration. The report that got the TIPS character right (Dudley, Macirowski, Richman, Strongin, and Youngdahl 1996, pp. 5–8) was actually written prior to the first U.S. TIPS issuance. Although the Goldman Sachs research report appears to represent the original discovery of the dual duration of TIPS, its lead does not appear to have been followed by others, even in the same organization's subsequent publications around that time. Having reintroduced the idea over the course of several Barclays Capital Global Inflation-Linked Bond Conferences, we are pleased to report that the concept of dual duration is now being much more widely used by practitioners and academics.
13. Of course, in practice, the leveraging asset is not riskless at any degree of leverage, much less as one approaches infinite positions. Figure 2 simply illustrates the general principle that leverage extends the potential reach of the strategy.
14. Another inflation source is inflation that is specific to a given industry, caused (for example) by the need for companies to attract and retain workers in a particularly competitive labor market. Such inflation, to the extent that it exceeds generalized or CPI (U.S. Consumer Price Index) inflation, cannot be hedged by TIPS.
15. Reasons exist to consider using a market-related, or economic, view of these liabilities (rather than a regulatory and actuarial view) when conducting economic tasks, including duration matching. The economic liability of the pension plan can be thought of as the fairly estimated present value of the complete stream of all cash flows that the pension plan will be obligated to pay, inclusive of all cash flows connected with the service provided by current and future employees through their expected date of retirement, irrespective of whether those cash flows are in the actuarially determined actual or projected benefit obligation. Furthermore, in the economist's view, these cash flows should be discounted at a rate consistent with the market-related (or beta) risk of the liabilities (not the expected return of the assets used to defease them, as the actuarial calculation would typically have it). See Treynor, Regan, and Priest (1976, p. 56); Bookstaber and Gold (1988); Michaud (1989, 1998); Ryan and Fabozzi (2002). The connection between economic liability and the problem of pension asset allocation policy is made by means of surplus optimization in Waring (2000a). A somewhat more complete treatment of the liability is in Waring (2004c).
16. Keep in mind that the relationship between salary inflation and generalized or, in the United States, CPI inflation is only approximate. To simplify the analysis, however, Goodman and Marshall assumed that these inflation rates were the same, as do we.
17. That is, for a 1 percent increase in *both* "anticipated" and "unanticipated" inflation, as defined by Goodman and Marshall, the present value of the liability would, given their other assumptions, decrease by 5.5 percent.
18. Actuarial and/or accounting measures of the liability, which use discount rates that react slowly to changes in market interest rates, do not necessarily move as described. But, of course, what counts for making economically sound decisions is the discounted PV, or *market* value, of the liability. This problem is best dealt with in an economic-liability context, which (by definition) uses a market-determined discount rate as described in Note 15.
19. TIPS strips (or, alternatively, leveraged TIPS) may be necessary to achieve certain combinations of inflation duration and real-interest-rate duration. In our example, if there are no coupon-paying plain TIPS with a real-interest-rate duration as large as 20.17 years, then investors who are unable to use strips or leverage must settle for an approximate hedge.
20. An inflation duration of zero means that the present value of the liability is unchanged no matter what the inflation rate—just what one would expect in a plan that is fully inflation indexed, because the higher nominal discount rate offsets the higher nominal payments to retirees. In practice, no COLA is so complete as to drive the inflation duration of the liability to exactly zero. For the present purpose, however, we assume it is exactly zero.
21. Or almost entirely. If the inflation duration of TIPS is negative but the inflation duration of the liability is exactly zero, a small position in nominal bonds is warranted.
22. This concept was introduced by a number of authors working independently at about the same time; for example, many of the articles by Leibowitz and his co-authors in the

first section of the Fabozzi (1992) book deal with the concept of surplus management (see also an early piece by Arnott and Bernstein 1988). The most widely known rendition of the math of surplus optimization is in Sharpe and Tint (1990), although Leibowitz and others have published variants. A current version is in Waring (2004c).

23. Duration is a multiperiod measure because it captures not only the risk to which a bond investor is exposed but also the way that risk can be expected to change over time.
24. The assets minus the liabilities are referred to as the surplus if positive and deficit if negative.
25. Waring (2004b) notes that there is a problem of zero or near-zero division unless one carefully defines the return and standard deviation of the surplus. Thus, the return of the surplus, R_S, is best defined relative to the market value by reference to the liability: $R_S = (S_1 - S_0/L_0) = [(A_1 - L_1) - (A_0 - L_0)/L_0] = (A_0/L_0)\, R_A - R_L$, where A = assets, L = liabilities, S = surplus = $A - L$, and 0 and 1 are time subscripts.
26. The modern literature on equities as an inflation hedge begins with Bodie (1976). A fascinating perspective is provided by Modigliani and Cohn (1979), who predicted the subsequent bull market as a correction of previous investors' errors in understanding the inflation-hedging properties of equities. The post-1979 literature on inflation and equities is extensive.
27. The inflation and real-interest-rate durations of equities can be estimated but without the precision that we associate with bond duration. A real rate duration of about 20 years and an inflation duration of about 4 years are estimates consistent with Leibowitz et al.
28. The precise formula is a little more complicated than implied in the text (payout must be at least 5 percent of the average asset values from the 12 month-ends in the fiscal year, and expenses associated with generating the investment return do not count toward payout, so they can push the payout rate to 5.2 percent or more).
29. Even if we acknowledge that some goods or services inflate at a rate higher than the general inflation rate as measured by the CPI, we do not have an instrument to hedge such high inflation rates.
30. In addition, the payout of a DB plan can sometimes be structured so that even survivors, such as a spouse, receive payments after the direct beneficiary's death.
31. An additional difference is that individuals can adjust both their contributions to and their withdrawals from the plan in response to changing investment returns and other circumstances. But if the individual's investment program gets in trouble, it is not secured by the resources of others as are DB plans, which are partially secured by the sponsor's balance sheet and the Pension Benefit Guaranty Corporation.
32. The assets that are relevant to a full asset/liability study for the individual include human capital (the discounted PV of future labor income); real estate and/or businesses; the discounted PV of expected DB pension income; the discounted PV of expected Social Security income; tax-deferred investment accounts, including DC pension accounts; taxable investment accounts; and cash and other miscellaneous assets. Liabilities include living expenses at an acceptable standard for the remainder of one's life; taxes; and mortgages and other debts. Anything remaining is "owner's equity," although one might classify a reserve for emergencies as a liability rather than equity. Typically, the only assets over which the individual has control, in the sense of deciding the asset-class mix (and the active strategy, if any, for beating the benchmark represented by that mix) are the tax-deferred and taxable investment accounts. Based on this analysis, one can arrive at an asset mix for these accounts that best diversifies the risks inherent in the other assets and that enhances the return of the overall portfolio.
33. Inflation-indexed life annuities would seem to be the most direct solution to the individual investor's problem as framed here. They enable the investor to exchange an investment balance into a stream of postretirement payments for life, thus mimicking a DB pension plan with a full COLA. The list of firms offering such plans is small, yet there are enough plans that investors can comparison-shop among them. For additional reading on inflation hedging for individual investors, see Hammond (1996), Brown, Mitchell, and Poterba (2000), and Warshawsky (2000).

References

Arnott, Robert D., and Peter L. Bernstein. 1988. "The Right Way to Manage Your Pension Fund." *Harvard Business Review* (January/February):95–102.

Bodie, Zvi. 1976. "Common Stocks as a Hedge against Inflation." *Journal of Finance*, vol. 31, no. 2 (May):459–470.

Bookstaber, Richard, and Jeremy Gold. 1988. "In Search of the Liability Asset." *Financial Analysts Journal*, vol. 44, no. 1 (January/February):70–80.

Brown, Jeffrey R., Olivia S. Mitchell, and James M. Poterba. 2000. "Mortality Risk, Inflation Risk, and Annuity Products." Working Paper 7812, National Bureau of Economic Research (July).

Dudley, William, Thomas Macirowski, Hayley Richman, Steve Strongin, and John Youngdahl. 1996. "Treasury Inflation-Protection Securities: A Useful Tool, but Not a Cure-All." Goldman Sachs Investment Research Report (October).

D'Vari, Ron, and Lal C. Chugh. 2002. "Treasury Inflation Protected Securities: New Perspectives on Their Significance to Portfolios." Working paper (July): www.rondvari.com/Business%20Journal_200206_D.pdf.

Dybvig, Philip H. 1999. "Using Asset Allocation to Protect Spending." *Financial Analysts Journal*, vol. 55, no. 1 (January/February):49–62.

Fabozzi, Frank J., ed. 1992. *Investing: The Collected Works of Martin L. Leibowitz*. Chicago, IL: Probus.

Fisher, Irving. 1965 (work originally done in 1930). *The Theory of Interest.* New York: Augustus M. Kelley.

Goodman, Laurie S., and William J. Marshall. 1988. "Inflation, Interest Rates, and Pension Liabilities." In *Asset Allocation: A Handbook of Portfolio Policies, Strategies, and Tactics*. Edited by Robert Arnott and Frank J. Fabozzi. Chicago, IL: Probus.

Hammond, P. Brett. 1996. "Real Bonds and Inflation Protection for Retirement." Research Dialogue Issue 47, TIAA-CREF (March).

Kothari, S.P., and Jay Shanken. 2000. "Asset Allocation with Conventional and Indexed Bonds." Presented at the Barclays Capital Inflation-Linked Bond Conference, Palm Beach, FL (14–15 February).

Leibowitz, Martin L., Eric H. Sorensen, Robert D. Arnott, and H. Nicholas Hanson. 1989. "A Total Differential Approach to Equity Duration." *Financial Analysts Journal,* vol. 45, no. 5 (September/October):30–37.

Macaulay, Frederick R. 1938. *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the United States since 1856.* New York: National Bureau of Economic Research.

Michaud, Richard. 1989. "Economic Surplus and Pension Management." Working paper, Merrill Lynch Capital Markets (February).

———. 1998. *Efficient Asset Management*. Boston, MA: Harvard Business School.

Modigliani, Franco, and Richard Cohn. 1979. "Inflation, Rational Valuation and the Market." *Financial Analysts Journal*, vol. 35, no. 2 (March/April):24–44.

Phoa, Wesley. 2001. "Quantitative Approaches to Inflation-Indexed Bonds." In *Handbook of Inflation-Indexed Bonds.* Edited by John Brynjolfsson and Frank J. Fabozzi. New Hope, PA: Frank J. Fabozzi Associates.

Ryan, Ronald J., and Frank J. Fabozzi. 2002. "Rethinking Pension Liabilities and Asset Allocation." *Journal of Portfolio Management*, vol. 28, no. 4 (Summer): 7–15.

Sharpe, William F., and Lawrence G. Tint. 1990. "Liabilities: A New Approach." *Journal of Portfolio Management*, vol. 16, no. 2 (Winter):5–10.

Siegel, Laurence B. 2001. "Inflation Hedging in a Low-Inflation World: The Plan Sponsor's Rationale." In *Handbook of Inflation-Indexed Bonds.* Edited by John Brynjolfsson and Frank J. Fabozzi. New Hope, PA: Frank J. Fabozzi Associates.

Treynor, Jack L., Patrick J. Regan, and William W. Priest, Jr. 1976. *The Financial Reality of Pension Funding under ERISA*. Homewood, IL: Dow Jones-Irwin.

Waring, M. Barton. 2000a. "Asset Allocation and Investment Policy: Beginning and Advanced Topics for the Serious Practitioner." Presented at the Asset Allocation Congress (organized by the Institute for International Research) in Palm Beach, FL (5 March); also presented at the 4 March 2001 conference.

———. 2000b. "TIPS and the Pension Plan." Presented at the Barclays Capital Inflation-Linked Bond Conference, Palm Beach, FL (14–15 February).

———. 2004a. "Liability-Relative Investing." *Journal of Portfolio Management*, vol. 30, no. 4 (Summer):8–20.

———. 2004b. "Liability-Relative Investing II." *Journal of Portfolio Management*, vol. 31, no. 1 (Fall).

———. 2004c. "Liability-Relative Strategic Asset Allocation Policies." In *The New World of Pension Fund Management*. Charlottesville, VA: CFA Institute (also available from the author as a working paper).

Warshawsky, Mark J. 2000. "Comments on 'The Role of Real Annuities and Indexed Bonds in an Individual Accounts Retirement Program' by Jeff Brown, Olivia Mitchell, and James Poterba." New York: TIAA-CREF (December): www.tiaa-crefinstitute.org/Publications/pubarts/pdfs/MW_pacmg-00.pdf.